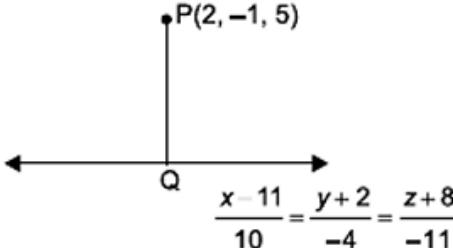


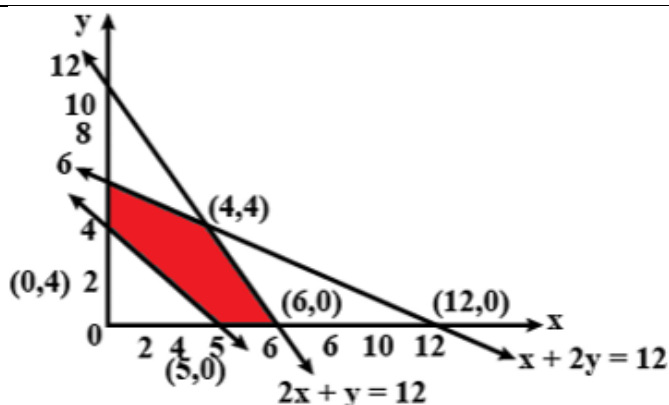


| SET – 1 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| 1       | a) -1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 2       | c) R                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 3       | a) $\begin{bmatrix} 5 & 14 \\ 0 & -7 \end{bmatrix}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 4       | d) 49y                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 5       | c) $\frac{(1 + \log x)^3}{3} + c$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 6       | a) $\frac{\pi}{6}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 7       | d) $\frac{5\vec{a}}{4}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 8       | a) $x = 12, y = 6$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 9       | c) 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 10      | b) 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 11      | c) $\frac{2}{1+x^2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 12      | c) Feasible solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 13      | d) $90^\circ$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 14      | b) (3, 2)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 15      | d) $\frac{1}{16}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 16      | d) $8\pi$ sq units                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 17      | a) $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 18      | d) 16                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 19      | (d) A is False but R is true.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 20      | (a) Both A and R are true and R is the correct explanation of A.                                                                                                                                                                                                                                                                                                                                                                                                                                                          |  |                                                                                                                                                                                                                                                                                                                                                                                                               |  |  |
| 21      | $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$<br>$\frac{dV}{dr} = 4\pi r^2$ and $\frac{dS}{dr} = 8\pi r$                                                                                                                                                                                                                                                                                                                                                                                                                    |  | $\frac{dV}{dS} = \frac{4\pi r^2}{8\pi r} = \frac{1}{2}r$<br>$\frac{dV}{dS} \Big _{r=2cm} = \frac{2}{2} = 1 \text{ cm}^3 / 1\text{cm}^2$                                                                                                                                                                                                                                                                       |  |  |
| 22      | <p>Ans: <math>a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)</math></p> <p><math>= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12}</math></p> <p><math>b = \tan^{-1}(\sqrt{3}) - \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}</math></p> <p>Now <math>a + b = \frac{11\pi}{12} - \frac{\pi}{3} = \frac{11\pi - 4\pi}{12} = \frac{7\pi}{12}</math></p> |  | <p>- OR -</p> <p>Ans: Given that <math>\sin^{-1}\left[k \tan\left(2 \cos^{-1} \frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}</math></p> <p><math>\Rightarrow k \tan\left(2 \cos^{-1}\left(\cos \frac{\pi}{6}\right)\right) = \sin \frac{\pi}{3}</math></p> <p><math>\Rightarrow k \tan \frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow k\sqrt{3} = \frac{\sqrt{3}}{2} \Rightarrow k = \frac{1}{2}</math></p> |  |  |

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                  |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 23 | $\cos \theta = \frac{\overline{PQ} \cdot \overline{PR}}{ \overline{PQ}   \overline{PR} }$ $\cos \theta = \frac{15+8+16}{\sqrt{9+16+16} \sqrt{25+4+16}}$ $\cos \theta = \frac{39}{\sqrt{41} \sqrt{45}}$ $\theta = \cos^{-1} \left( \frac{39}{3\sqrt{205}} \right) = \cos^{-1} \left( \frac{13}{\sqrt{205}} \right)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | <p>- OR -</p> $\frac{\overline{PQ} + \overline{PR}}{2} = \frac{(-3i+4j+4k) + (-5i+2j+4k)}{2}$ $\frac{-8i+6j+8k}{2} = 4i+3j+4k$ <p>The length of median = <math> 4i+3j+4k  = \sqrt{41}</math></p> |
| 24 | <p>Ans. General point on the line <math>\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda</math> is</p> <p><math>Q(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)</math> ... (i)</p> <p><math>P(2, -1, 5)</math></p>  <p>Direction ratios of PQ are <math>10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5</math><br/>i.e. <math>10\lambda + 9, -4\lambda - 1, -11\lambda - 13</math></p> <p>If PQ is perpendicular to the given line, then</p> $10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$ $\Rightarrow 237\lambda = -237 \quad \lambda = -1$ <p>Substituting in (i), we get the foot of perpendicular as Q(1, 2, 3).</p> <p>Length of perpendicular PQ = <math>\sqrt{(2-1)^2 + (-1-2)^2 + (5-3)^2} = \sqrt{1+9+4} = \sqrt{14}</math></p>                                                                                                   |                                                                                                                                                                                                  |
| 25 | $\Rightarrow x \sin(a+y) = -\sin a \cos(a+y)$ $\Rightarrow x = \frac{-\sin a \cos(a+y)}{\sin(a+y)} \Rightarrow x = -\sin a \cot(a+y)$ <p>Differentiating with respect to y, we get <math>\frac{dx}{dy} = -\sin a [-\csc^2(a+y)] \cdot \frac{d}{dy}(a+y)</math></p> $= -\sin a [-\csc^2(a+y)] \cdot (0+1) = \frac{\sin a}{\sin^2(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ <p>- OR -</p> <p>Let <math>I = \int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}} = \int_0^{\pi/2} \frac{dx}{1 + \frac{\sqrt{\sin x}}{\sqrt{\cos x}}} = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx</math> ... (i)</p> <p>By the property, <math>\int_0^a f(x) dx = \int_0^a f(a-x) dx</math>, we get</p> $I = \int_0^{\pi/2} \frac{\sqrt{\cos(\frac{\pi}{2}-x)}}{\sqrt{\cos(\frac{\pi}{2}-x)} + \sqrt{\sin(\frac{\pi}{2}-x)}} dx = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ ... (ii) <p>Adding (i) and (ii), we get</p> |                                                                                                                                                                                                  |

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|    | $2I = \int_0^{\pi/2} \left[ \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right] dx = \int_0^{\pi/2} 1 \cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 26 | $ x+2  = \begin{cases} x+2, & x \geq -2 \\ -(x+2), & x < -2 \end{cases}$ $\int_{-5}^5  x+2  dx = \int_{-5}^{-2} -(x+2) dx + \int_{-2}^5 (x+2) dx$ $= -\left(\frac{x^2}{2} + 2x\right)_{-5}^{-2} + \left(\frac{x^2}{2} + 2x\right)_{-2}^5$ $= -\left(\frac{4}{2} - 4 - \frac{25}{2} + 10\right) + \left(\frac{25}{2} + 10 - \frac{4}{2} + 4\right)$                                                                                                                                                                                                                                                                                                                                | $= -\left(\frac{4-8-25+20}{2}\right) + \left(\frac{25+20-4+8}{2}\right) = \frac{9}{2} + \frac{49}{2} = 29$                                                                                                                                                                                                                                                                                                                                                         |
| 27 | <p><math>S = \{BB, BG, GB, GG\}</math></p> <p>(i) A: at least one of the children is a boy = BB, BG, GB<br/> B: both are boys = BB<br/> <math>A \cap B</math>: BB<br/> both boys when at least one of the children is a boy</p> $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$                                                                                                                                                                                                                                                                                                                                                               | <p>(ii) A: the elder child is a boy = BB, BG<br/> B: both are boys = BB<br/> <math>A \cap B</math>: BB<br/> Probability of the elder child is a boy.</p> $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$ <p style="text-align: center;">- OR -</p>                                                                                                                                                                             |
|    | <p>Ans: Bag I: 3 red + 4 black,<br/> Bag II: 4 red + 5 black<br/> <b>Case I</b> : when ball transferred is black.</p> $P(B/I) = \frac{4}{7}$ <p>Total balls in bag II are 4 red + 6 black;</p> $P(R/II) = \frac{4}{10}$ <p>Probability in this case = <math>\frac{4}{7} \times \frac{4}{10}</math>.</p>                                                                                                                                                                                                                                                                                                                                                                           | <p><b>Case II</b>: When ball transferred is red.</p> $P(R/I) = \frac{3}{7}$ <p>Total balls in bag II are 5 red + 5 black,</p> $P(R/II) = \frac{5}{10}$ <p>Probability in this case = <math>\frac{3}{7} \times \frac{5}{10}</math></p> <p>Using Bayes' Theorem, probability that the ball transferred is black</p> $= \frac{\frac{4}{7} \times \frac{4}{10}}{\frac{4}{7} \times \frac{4}{10} + \frac{3}{7} \times \frac{5}{10}} = \frac{16}{16+15} = \frac{16}{31}$ |
| 28 | $\int \frac{1}{9x^2 + 6x + 5} dx = \frac{1}{9} \int \frac{1}{x^2 + \frac{6}{9}x + \frac{5}{9}} dx$ $= \frac{1}{9} \int \frac{1}{x^2 + \frac{2}{3}x + \frac{5}{9} + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2} dx = \frac{1}{9} \int \frac{1}{\left(x + \frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} dx, \text{ putting } x + \frac{1}{3} = t \Rightarrow dx = dt$ $= \frac{1}{9} \int \frac{1}{t^2 + \left(\frac{2}{3}\right)^2} dt = \frac{1}{9} \cdot \frac{1}{\frac{2}{3}} \tan^{-1}\left(\frac{t}{2/3}\right) + C = \frac{1}{6} \tan^{-1}\left[\frac{3\left(x + \frac{1}{3}\right)}{2}\right] + C = \frac{1}{6} \tan^{-1}\left(\frac{3x+1}{2}\right) + C$ |                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |

29



| Corner points | $Z = 600x + 400y$ |
|---------------|-------------------|
| (0,4)         | 1600 minimum      |
| (0,6)         | 2400              |
| (4,4)         | 4000 maximum      |
| (6,0)         | 3600              |
| (5,0)         | 3000              |

30

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$$

$$= \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \tan^{-1}(\sin x) dx$$

$$\text{let } \sin x = t \Rightarrow \cos x dx = dt$$

$$\text{When } x = 0, t = 0, \text{ when } x = \frac{\pi}{2}, t = 1$$

$$\Rightarrow I = 2 \int_0^1 t \tan^{-1}(t) dt \quad \dots(i)$$

$$\int t \cdot \tan^{-1} t dt =$$

$$= \tan^{-1} t \cdot \int t dt - \int \left\{ \frac{d}{dt} (\tan^{-1} t) \int t dt \right\} dt$$

$$= \tan^{-1} t \cdot \frac{t^2}{2} - \int \frac{1}{1+t^2} \cdot \frac{t^2}{2} dt$$

$$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \int \frac{t^2 + 1 - 1}{1+t^2} dt$$

$$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \int 1 dt + \frac{1}{2} \int \frac{1}{1+t^2} dt$$

$$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \cdot t + \frac{1}{2} \tan^{-1} t$$

$$\Rightarrow \int_0^1 t \cdot \tan^{-1} t dt = \left[ \frac{t^2 \cdot \tan^{-1} t}{2} - \frac{t}{2} + \frac{1}{2} \tan^{-1} t \right]_0^1$$

$$= \frac{1}{2} \left[ \frac{\pi}{4} - 1 + \frac{\pi}{4} \right] = \frac{1}{2} \left[ \frac{\pi}{2} - 1 \right] = \frac{\pi}{4} - \frac{1}{2}$$

$$I = 2 \left[ \frac{\pi}{4} - \frac{1}{2} \right] = \frac{\pi}{2} - 1$$

31

We have,  $(1 + e^{2x}) dy + (1 + y^2) e^x dx = 0$  and given that  $y = 1$ , when  $x = 0$

$$\therefore \frac{dy}{dx} = \frac{-(1 + y^2)e^x}{1 + e^{2x}} \Rightarrow \frac{dy}{-(1 + y^2)} = \frac{e^x dx}{1 + e^{2x}}$$

Integrating both sides, we get

$$-\int \frac{dy}{1 + y^2} = \int \frac{e^x dx}{1 + e^{2x}} \Rightarrow -\tan^{-1} y = \int \frac{e^x dx}{1 + (e^x)^2}$$

$$\Rightarrow -\tan^{-1} y = \int \frac{dt}{1 + t^2} \quad [\text{Putting } e^x = t \Rightarrow e^x dx = dt]$$

$$\Rightarrow -\tan^{-1} y = \tan^{-1}(t) + C \Rightarrow -\tan^{-1} y = \tan^{-1}(e^x) + C$$

Put  $x = 0, y = 1$  in (i), we get

$$-\tan^{-1} 1 = \tan^{-1}(e^0) + C \Rightarrow -\frac{\pi}{4} = \frac{\pi}{4} + C \Rightarrow C = -\frac{\pi}{2}$$

Putting the value of  $C$  in (i), we get

$$-\tan^{-1} y = \tan^{-1}(e^x) - \frac{\pi}{2} \Rightarrow \frac{\pi}{2} = \tan^{-1}(e^x) + \tan^{-1} y$$

Hence,  $\tan^{-1}(e^x) + \tan^{-1} y = \frac{\pi}{2}$  is the required solution.

- OR -

Comparing it with  $\frac{dy}{dx} + Py = Q$ , we get  $P = 2 \tan x$ ,  $Q = \sin x$

$$\therefore \text{IF} = e^{\int 2 \tan x dx} = e^{2 \log \sec x} = e^{\log \sec^2 x} = \sec^2 x \quad [\because e^{\log z} = z]$$

Hence, general solution is  $y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x dx + C$ .

$$y \cdot \sec^2 x = \int \sec x \cdot \tan x dx + C \Rightarrow y \cdot \sec^2 x = \sec x + C \Rightarrow y = \cos x + C \cos^2 x$$

Putting  $y = 0$  and  $x = \frac{\pi}{3}$ , we get  $0 = \cos \frac{\pi}{3} + C \cdot \cos^2 \frac{\pi}{3}$

$$\Rightarrow 0 = \frac{1}{2} + \frac{C}{4} \Rightarrow C = -2$$

$\therefore$  Required solution is  $y = \cos x - 2 \cos^2 x$ .

32

$$x + y + z = 6$$

$$y + 3z = 11$$

$$x + z = 2y \text{ or } x - 2y + z = 0$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ \& } B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$A = 1(1 + 6) - 0 + 1(3 - 1) = 9$$

$$\text{Hence, } \text{adj}(A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\text{Thus, } A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} :$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1, y = 2, z = 3$$

- OR -

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= 1(2 - 2) - 2(3 + 4) - 3(-3 - 4)$$

$$= -14 + 21 = 7$$

$$\therefore \text{adj } A = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The given system of equations is  $x + 2y - 3z = 6$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$

The system of equations can be written as  $AX = B$

$$\text{where } A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$\therefore A^{-1}$  exists, so system of equations has a unique solution given by  $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$$\Rightarrow x = 1, y = -5, z = -5$$

33

Ans: Given:  $y^2 = 2x$

$$y = x - 4$$

Required area is OABCO

from (1) and (2),  $(x - 4)^2 = 2x$

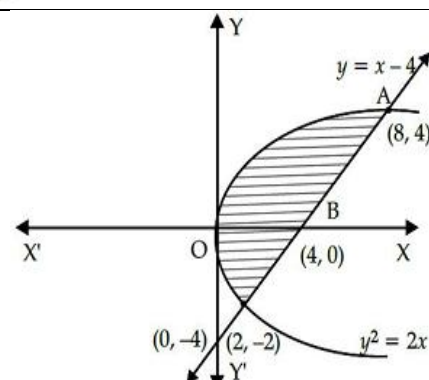
$$\Rightarrow x^2 - 10x + 16 = 0$$

$$\Rightarrow (x - 8)(x - 2) = 0$$

$$\Rightarrow x = 8 \text{ and } x = 2$$

$\therefore$  Intersection points  $(2, -2)$  and  $(8, 4)$

$$\text{Area} = \left[ \frac{y^2}{2} + 4y - \frac{y^3}{6} \right]_{-2}^4 = \left( 8 + 16 - \frac{32}{3} - 2 + 8 - \frac{4}{3} \right) = 30 - 12 = 18 \text{ unit}^2$$



|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 34 | <p>Ans: Relation <math>R</math> on <math>N \times N</math> is given by<br/> <math>(a, b) R(c, d) \Leftrightarrow ad(b+c) = bc(a+d)</math><br/> For reflexive:<br/> For <math>(a, b) \in N \times N</math><br/> <math>(a, b) R(a, b) \Rightarrow ab(b+a) = ba(a+b)</math><br/> true in <math>N</math><br/> Hence, reflexive<br/> For symmetric:<br/> For <math>(a, b), (c, d) \in N \times N</math><br/> <math>(a, b) R(c, d) \Rightarrow ad(b+c) = bc(a+d)</math><br/> <math>\Rightarrow cb(d+a) = da(c+b)</math> ( <math>\times</math> )<br/> <math>\Rightarrow (c, d) R(a, b) \forall (a, b), (c, d) \in N \times N</math>.<br/> Hence, symmetric<br/> For transitive:<br/> For <math>(a, b), (c, d), (e, f) \in N \times N</math><br/> Let <math>(a, b) R(c, d)</math> and <math>(c, d) R(e, f)</math><br/> <math>\Rightarrow ad(b+c) = bc(a+d)</math><br/> <math>\Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}</math></p>          | <p>and <math>cf(d+e) = de(c+f)</math><br/> <math>\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}</math><br/> <math>\Rightarrow \frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}</math><br/> <math>\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}</math><br/> <math>af(e+b) = be(f+a)</math><br/> <math>\Rightarrow af(b+e) = be(a+f)</math><br/> <math>\Rightarrow (a, b) R(e, f)</math><br/> As <math>(a, b) R(c, d), (c, d) R(e, f)</math><br/> <math>\Rightarrow (a, b) R(e, f)</math> Hence, transitive.<br/> As relation <math>R</math> is reflexive, symmetric and transitive. Hence, <math>R</math> is an equivalence relation.</p>                           |
| 35 | <p>i) <math>\vec{a}_1 = i + 2j + 3k; \vec{b} = i - 3j + 2k</math><br/> <math>\vec{a}_2 = 4i + 5j + 6k</math> Both lines are parallel.<br/> <math>\vec{a}_2 - \vec{a}_1 = 3i + 3j + 3k</math><br/> <math>\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} i &amp; j &amp; k \\ 1 &amp; -3 &amp; 2 \\ 3 &amp; 3 &amp; 3 \end{vmatrix}</math><br/> <math>= i(-9-6) - j(3-6) + k(3-(-9))</math><br/> <math>= -15i + 3j + 12k</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | <p>Shortest distance = <math>\frac{ \vec{b} \times (\vec{a}_1 - \vec{a}_2) }{ \vec{b} }</math><br/> <math>= \frac{ -15i + 3j + 12k }{ i - 3j + 2k }</math><br/> <math>= \frac{\sqrt{225+9+144}}{\sqrt{1+9+4}} = \frac{\sqrt{378}}{\sqrt{14}}</math><br/> <math>= \frac{3\sqrt{3}\sqrt{14}}{\sqrt{14}} = 3\sqrt{3}</math> - OR</p>                                                                                                                                                                                                                                                                                                                                                                                                                          |
|    | <p>(i)<br/> Ans: Given that <math>\vec{a} = \hat{i} - \hat{j} + 7\hat{k}</math> and <math>\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}</math><br/> <math>\therefore \vec{a} + \vec{b} = \hat{i} - \hat{j} + 7\hat{k} + 5\hat{i} - \hat{j} + \lambda\hat{k} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}</math><br/> and <math>\vec{a} - \vec{b} = \hat{i} - \hat{j} + 7\hat{k} - 5\hat{i} + \hat{j} - \lambda\hat{k} = -4\hat{i} + (7-\lambda)\hat{k}</math><br/> Now, <math>\vec{a} + \vec{b}</math> and <math>\vec{a} - \vec{b}</math> are perpendicular vectors<br/> <math>\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0</math><br/> <math>\Rightarrow (6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}) \cdot (-4\hat{i} + (7-\lambda)\hat{k}) = 0</math><br/> <math>\Rightarrow -24 + 0 + (7+\lambda)(7-\lambda) = 0</math><br/> <math>\Rightarrow -24 + 49 - \lambda^2 = 0 \Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5</math></p> | <p>(ii)<br/> <math>\vec{AB} = (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = -\hat{i} - 2\hat{j} - 6\hat{k}</math><br/> <math>\Rightarrow  \vec{AB}  = \sqrt{1+4+36} = \sqrt{41}</math><br/> <math>\vec{BC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k}) = 2\hat{i} - \hat{j} + \hat{k}</math><br/> <math>\Rightarrow  \vec{BC}  = \sqrt{4+1+1} = \sqrt{6}</math><br/> and <math>\vec{AC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = \hat{i} - 3\hat{j} - 5\hat{k}</math><br/> <math>\Rightarrow  \vec{AC}  = \sqrt{1+9+25} = \sqrt{35}</math><br/> <math>\therefore  \vec{AB} ^2 =  \vec{AC} ^2 +  \vec{BC} ^2</math><br/> Hence, <math>ABC</math> is a right angled triangle.</p> |
| 36 | <p>Ans: (i) Since, <math>\sum P(x) = 1</math><br/> <math>\therefore k + 2k + 3k + 0 = 1</math><br/> <math>\Rightarrow 6k = 1 \Rightarrow k = 1/6</math><br/> (ii) From (i), <math>k = 1/6</math><br/> <math>\therefore P(x=2) = 3k = 3/6 = 1/2</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | <p>(iii) If a students has study time at least 1 hr then either he/she has studies for 1 hour or 2 hour.<br/> <math>\therefore</math> Required Probability = <math>P(x=1) + P(x=2) = 2k + 3k = 5k = \frac{5}{6}</math><br/> Mean = <math>\sum xP(x) = 0 \times \frac{1}{6} + 1 \times \frac{2}{6} + 2 \times \frac{3}{6} = 0 + \frac{2}{6} + 1 = \frac{1}{3} + 1 = \frac{4}{3}</math></p>                                                                                                                                                                                                                                                                                                                                                                  |

37 Ans: (a)  $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$   
 $\therefore$  For 2000,  $t = 0$   
 $\therefore V(0) = 0 - 0 + 0 - 2$   
 Since, number of vehicles cannot be negative.  
 Therefore, the given function cannot be used to estimate number of vehicles in the year 2000.

(b)  $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$   

$$V'(t) = \frac{3}{5}t^2 - 5t + 25 = \frac{3}{5}\left[t^2 - \frac{25}{3}t + \frac{125}{3}\right]$$

$$= \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 - \frac{625}{36} + \frac{125}{3}\right] = \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 + \frac{875}{36}\right]$$

$V'(t) > 0$  for any value of  $t$   
 $\therefore$  The given function  $V(t)$  is an increasing function.

38 Ans: (i) Required relation is given by  $2x + y = 200$ .  
 (ii) Area of garden as a function of  $x$  can be represented as  $A(x) = xy = x(200 - 2x)$   
 $= 200x - 2x^2$   
 (iii)  $A(x) = 200x - 2x^2$   
 $\Rightarrow A'(x) = 200 - 4x$   
 For the area to be maximum  $A'(x) = 0 \Rightarrow 200 - 4x = 0$   
 $\Rightarrow x = 50$  ft.  

**OR**

 (iii) Maximum area of the garden  $= 200(50) - 2(50)^2$   
 $= 10000 - 5000 = 5000$  sq. ft